CHARACTERISATION OF PLANAR WYE SHAPED RESONATORS FOR USE IN CIRCULATOR HARDWARE

by W.T. Nisbet, Racal-MESL Limited, Lochend Ind. Estate, Newbridge, Midlothian, Scotland.

J. Helszajn, Dept. of Electrical Engineering, Heriot Watt University, Edinburgh, Scotland.

Abstract

An important class of three port junction circulators consist of a network of six transmission lines alternatively terminated in 50Ω lines and magnetic walls (open-circuits). In general, this network has threefold symmetry, since consecutive transmission lines differ both in impedance and electrical length. Properly adjusted, it exhibits a frequency response normally associated with a quarter-wave coupled junction circulator. For circulators, for which the in-phase eigennetwork may be represented by an ideal short circuit, the equivalent circuit is a 1-port network which may be formed from the definition of the constituent resonator. This feature is used in this paper to study the equivalent circuit of junction circulators using microstrip WYE resonators on garnet substrates.

Introduction

A widely used, but poorly understood, commercial three port stripline circulator geometry consists of a junction of three open-circuited transmission lines symmetrically fed by three additional transmission lines terminated in matched loads, the complete assembly being embedded between two ferrite disks. This circuit arrangement is illustrated in figure (1). An important feature of this circuit is that it may be adjusted to display the type of frequency characteristic normally associated with a conventional quarter-wave coupled junction, but with physical dimensions of the order of a directly coupled device.

The network in figure (1) may be understood by assuming that the junction resonator consists of three symmetrically connected open-circuited stubs [1]. The standing wave solution in this resonator is shown in figure (2). Such a resonator may be symmetrically coupled to 50 ohm lines in either of the two ways illustrated in figure (3) a and b. If it is connected in the manner depicted in figure (3) a, it may be viewed as a conventional quarter-wave coupled circulator, for which standard quarter-wave matching theory applies. If the junction is coupled in the way indicated in figure (3) b, the network must be interpreted as a junction of three resonant stubs terminated in 50 ohm resistors symmetrically loaded by three open-circuited transmission lines. Whereas, a circulator may be defined by either of the above two boundary conditions, the situation described in figure (3) a forms the main topic of this paper in that it reflects commercial practice.

Circulators using planar WYE shaped resonators may be analysed by forming a 6 x 6 impedance matrix for the central non-resonant disk region defined by the three open-circuited stubs of the resonator circuit and the three coupling intervals of the circulator terminals. The boundary conditions at the terminals of the three stubs are subsequently used to reduce the 6 x 6 matrix to the required 3 x 3 impedance matrix of the circulator. However, comparison with experiment indicates that unless a similar number of modes are retained in the disk and stub regions, the entries of the latter matrix may not be reliable over all ranges of physical variables 7 .

The approach used in this study is therefore based on the one port equivalent circuit of a junction circulator in terms of its susceptance slope parameter and gyrator admittance. This representation is applicable provided that the in-phase mode can be represented by a frequency independent short circuit, and is useful for characterising devices with moderate specifications, (25% bandwidth at 25 dB return loss points). When the in-phase mode is idealised in this manner, the susceptance slope parameter and split frequencies of the counterrotating modes of the junction may be obtained by forming the constituent resonator [2,3,4]. This latter circuit is a 1-port reactive network defined by placing open-circuit boundary conditions at two of the three ports at the resonator terminals. The 3 x 3 impedance matrix is now reduced to a 1 x 1 scalar problem.

This report presents a study of the frequency splitting between the counterrotating modes and the susceptance slope parameter of microstrip WYE shaped resonators, based on a measurement of the reflection coefficient of the corresponding constituent resonator. Since the gyrator conductance is related to the split frequencies and the susceptance slope parameter by the universal circulation equation, the experimental characterisation of the circulator, in terms of its one port equivalent circuit, is complete.

Constituent Resonator of Junction Circulators

The development of the three port junction circulator involves the adjustment of the reflection coefficient of two counter-rotating modes with respect to that of an in-phase mode. In circulators for which the in-phase reflection coefficient can be idealised by a frequency independent short-circuit boundary condition, the junction can be represented by the 1-port equivalent circuit depicted in figure (4). Here, G is the gyrator conductance and B' is the susceptance slope parameter of the junction. The latter two quantities are related to the resonant frequencies of the single counter-rotating modes $\omega_{\pm 1}$ by the universal circulation equation [5].

$$G = \sqrt{3} \,.\, B' \, \frac{\omega_{+1} - \omega_{-1}}{\omega_0} \tag{1}$$

where ω_0 is the centre frequency of the circulator, and its equivalent circuit.

The loaded Q-factor of the junction is defined in terms of the split frequencies by

$$\frac{1}{Q_L} = \frac{G}{B'} = \sqrt{3} \, \frac{\omega_{+1} - \omega_{-1}}{\omega_0} \tag{2}$$

The universal circulation equation is only defined for circulators which rely on symmetric splitting for their operation. Clearly a knowledge of any two of the three variables in (1) completes the characterisation of the circulator in terms of the one port equivalent circuit in figure (4).

The susceptance slope parameter and frequency characteristics of junction circulators may be determined by assuming that the demagnetised junction consists of the connection of three constituent resonators [2,3,4] ; each resonator being mutually coupled to the others [2]. The constituent resonator is defined by placing open circuits at the resonator terminals of two of the three circulator ports. The constituent WYE resonator is illustrated in figure (5).

The relationship between the susceptance slope parameter of the constituent resonator and that of the circulator is a constant which may be obtained from a comparison of their respective input susceptances.

The voltage-current relationship for a reciprocal, symmetrical three port junction is,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{12} \\ Z_{12} & Z_{11} & Z_{12} \\ Z_{12} & Z_{12} & Z_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{3}$$

The constituent resonator is defined by applying magnetic walls at ports 2 and 3 of the reciprocal 3-port network. Thus

$$I_2 = I_3 = 0 \tag{4}$$

The input impedance of the constituent resonator is readily expressed as

$$Z_{in} = Z_{11} = \frac{2Z_1 + Z_o}{3} = \frac{2Z_1}{3} \tag{5}$$

Z_1 is the impedance eigenvalue of the degenerate (demagnetised) counter-rotating modes, Z_o is the impedance eigenvalue of the in-phase mode, and is taken as zero in accordance with the idealisation of the in-phase eigen-network as a short-circuit boundary condition.

The input admittance (susceptance) of the constituent resonator is given by inverting (5) as,

$$Y_{in} = \frac{3Y_1}{2} \tag{6}$$

Y_1 is the degenerate admittance eigenvalue of the counter-rotating eigen-networks, with

$$Y_1 = \frac{1}{Z_1} \tag{7}$$

The voltage-current relation for a nonreciprocal 3-port junction is

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{13} & Z_{11} & Z_{12} \\ Z_{12} & Z_{13} & Z_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{8}$$

The boundary conditions of an ideal circulator are

$$V_3 = I_3 = 0 \tag{9}$$

Combining the preceding equation leads to the following standard form for the input impedance

$$Z_{in} = Z_{11} - \frac{Z_{12}^2}{Z_{13}} \tag{10}$$

If the in-phase eigen-network is idealised by a frequency independent short circuit, the input admittance may be expressed as,

$$Y_{in} = \frac{1}{Z_{in}} = \frac{Y_{+1} + Y_{-1}}{2} + j\sqrt{3}\,\frac{Y_{+1} - Y_{-1}}{2} \tag{11}$$

$Y_{\pm 1}$ are the magnetised matrix eigenvalues, which have the following in the vicinity of the operating frequency of the junction [6].

$$Y_{\pm 1} = Y_1 \mp j\,\frac{Y_o}{\sqrt{3}} \tag{12}$$

Y_o is the characteristic admittance of the external transmission lines.

Combining the two preceding equations leads to

$$Y_{in} = G + jB = Y_o + Y_1 \tag{13}$$

A comparison of the input susceptance of the constituent resonator in (6) and the junction in (13) indicates that they are related by

$$B = \frac{2}{3} B_o \tag{14}$$

B is the susceptance of the circulator and B_o that of the constituent resonator.

The susceptance slope parameter of the constituent resonator is defined in the usual way by

$$B'_o = \frac{\omega_o}{2} \left.\frac{\partial B_o}{\partial \omega}\right|_{\omega = \omega_o} \tag{15}$$

The above expressions indicate that the susceptance slope parameters of the constituent resonator and that of the circulator are related by

$$B' = \frac{2}{3} B'_o \tag{16}$$

Thus the susceptance slope parameter of junction circulators (for which the in-phase eigenvalue can be idealised by a short-circuit boundary condition) may be obtained from an analysis or measurement of the constituent resonator.

Experimental Characterisation of Circulators Using WYE Resonators

The parameters of the one port equivalent circuit of junction circulators may be characterised by an e.m. analysis of the junction or by measurement. The approach used in this report relies on a measurement of the reflection coefficient of the constituent resonator. The relationship between the measured reflection coefficient and the susceptance slope parameter is derived in this section.

The input admittance of the constituent resonator may be written in terms of its electrical length θ as,

$$Y_{in} = jB_o = jY_o \cot\theta \tag{17}$$

Forming the susceptance slope parameter, defined in (15), in terms of θ leads to,

$$B'_o = Y_o \frac{\omega_o}{2} \operatorname{cosec}^2\theta_o \cdot \left.\frac{\delta\theta}{\delta\omega}\right|_{\omega = \omega_o} = Y_o \frac{\omega_o}{2} \operatorname{cosec}^2\theta_o \cdot \left.\frac{\Delta\theta}{\Delta\omega}\right|_{\omega=\omega_o} \tag{18}$$

assuming a linear relationship between the input susceptance and frequency.

Setting $\theta_o = \pi/2$ at $\omega = \omega_o$, the above equation becomes,

$$B'_o = Y_o \frac{\omega_o}{2} \cdot \frac{\theta - \pi/2}{\omega - \omega_o} \tag{20}$$

The above expression indicates that the susceptance slope parameter may be defined experimentally from a measurement of the reflection phase of the constituent resonator in the vicinity of its resonant frequency.

Although the constituent resonator is ideally a reactive network, the finite dissipation in, and radiation from the circuit will cause the amplitude of the reflection coefficient to depart from the ideal value of unity. The maximum effect will be observed at the centre frequency of the resonator, enabling a convenient measurement of this parameter.

Susceptance Slope Parameter of Circulators Using Planar WYE Resonators

The theoretical susceptance slope parameter of circulators using WYE resonators with perfect magnetic wall boundary conditions has been evaluated in reference (7) using a mode matching analysis of the constituent WYE resonator. One mode was retained in each region. The result is,

$$B = \frac{4\pi n}{3}\sqrt{\frac{\varepsilon_o \varepsilon_f}{\mu_o \mu_d}}\,\frac{R_i}{h}\left[\frac{3}{2} \cdot \frac{\sin^2\psi_s . \theta_s . \sec^2\theta_s}{\pi\psi_s} + \frac{kR_i^2 - 1}{2kR_i} + kR_i \cdot \left(\frac{3\sin^2\psi_s}{\pi\psi_s} . \tan\theta_s\right) . \left(\frac{3\sin^2\psi_s}{\pi\psi_s} . \tan\theta_s + \frac{1}{kR_i}\right)\right] \tag{21}$$

where n=1 for a stripline and n=2 for a microstrip resonator, h is the resonator thickness in metres and,

$$\theta_s = k(R_o - R_i) \tag{22}$$

$$k = \frac{2\pi}{\lambda_o} \sqrt{\mu_d \varepsilon_f} \tag{23}$$

λ_o is the free space wavelength in metres, ε_f is the dielectric constant and μ_d the demagnetised permeability of the garnet material defined by,

$$\mu_d = \frac{1}{3} + \frac{2}{3}\left[1 - \left(\frac{\gamma M_o}{\mu_o \omega}\right)^2\right]^{\frac{1}{2}} \tag{24}$$

M_o is the saturation magnetisation in Tesla, γ is the gyromagnetic ratio (=2.21 (rad/sec)/(A/m) and ω is the radian resonant frequency. The above expression indicates that the susceptance slope parameter is independent of coupling angle. For a given resonator shape, defined by R_i/R_o and ψ_s, B' may only be adjusted by altering the resonator thickness. In the case of microstrip circuits, where the substrate thickness is set by the system requirement, this leads to a constant susceptance slope parameter device. At R_i/R_o = 1, B' corresponds to that of a disk resonator.

The experimental hardware consisted of a range of microstrip constituent resonators on garnet substrates with dimensions 25 x 25 x 0.635 mm, a saturation magnetisation of 0.1200 T, and a dielectric constant of 15.2. The dimensions were chosen to give values of R_i/R_o and ψ_s in the range 0.2 - 0.8. R_i/R_o and ψ_s are defined in figure (1). The radius of the open-circuited stubs, R_o was calculated for each combination of R_i/R_o and ψ_s to give a resonant frequency of 9 GHz with the help of (25) given below. The phase of the reflection coefficient was measured using an accuracy enhanced automatic network analyser, and a typical response is depicted in figure (6). The accompanying print out of the response enabled the susceptance slope parameter to be accurately calculated using (20).

One convenient, concise method of representing the relationship between the susceptance slope parameter and the resonator geometry is to form the product $B'h/R_o\ (\mu_o\mu_d/\varepsilon_o\varepsilon_f)^{\frac{1}{2}}$ and plot against the stub coupling angle $\psi_s$for parametric values of R_i/R_o. The experimental points are summarised in this manner in figure (8). No attempt was made to characterise the fringing of the circuit to allow a comparison with the theoretical susceptance slope parameter in (21).

Cut-off Wave Number of Microstrip WYE Resonators

The resonant frequency of the microstrip constituent resonators has also been measured, and the experimental cut-off wave number, kR_o, with k defined in (23) calculated. The results are summarised in figure (9).

The theoretical cut-off wave number for WYE resonators with perfect magnetic wall boundary conditions is given in reference (7), as

$$kR_o = kR_i + \tan^{-1}\left[\frac{\pi\psi_s}{3\sin^2\psi_s}\,\frac{J_1'(kR_i)}{J_1(kR_i)}\right] \tag{25}$$

The theoretical cut-off wave number is superimposed on the experimental points in figure (9). The discrepancy between theory and practice is due to the imperfect magnetic wall boundary conditions at the periphery of the experimental constituent resonators.

The cut-off wave number at R_i/R_o = 1 corresponds to that of microstrip disk resonators [8].

Experimental Split Frequencies of Microstrip WYE Resonators

In order to complete the characterisation of the universal circulation equation, it is necessary to have a knowledge of the split frequencies of the magnetised resonator in addition to a knowledge of the susceptance slope parameter.

The split frequencies of microstrip WYE resonators were characterised experimentally by monitoring the variation of the amplitude of the reflection coefficient with applied magnetic field [9]. The experimental split frequencies are summarised in figures (10) to (13). Each illustration corresponds to a set of resonators with a fixed stub coupling angle and parametric values of R_i/R_o. Although the experimental resonators were designed to operate at 9 GHz, with the help of (25), the differing fringing characteristics of each shape altered the resonant frequency accordingly, as depicted in figure (9). A consideration of figures (10) to (13) indicates that the splitting increases as $R_i/R_o \to 1$ and as ψ_s increases.

The point of inflexion of the lower branch of each set of split frequencies corresponds to the applied field at which the garnet material saturates. Since the substrate is relatively thin, 0.635 mm, the axial demagnetising factor is approximately unity, and the applied field required for saturation is,

$$H_{o,sat} = \frac{M_o}{\mu_o} = 95.5 \text{ kA/m.} \tag{26}$$

Inspection of figures (10) thru (13) indicates that this is satisfied experimentally.

Conclusions

The equivalent circuit of junction circulators for which the in-phase eigennetwork may be idealised by a frequency independent short circuit boundary condition is fully described in terms of its susceptance slope parameter and the split frequencies of the magnetised resonator. A one-port circuit which allows both quantities to be characterised is the constituent resonator of the junction. It is used in this report to study the equivalent circuit of junction circulators using WYE resonators. Having defined the equivalent circuit experimentally, the synthesis of directly and transformer coupled devices proceeds in the standard manner [10].

References

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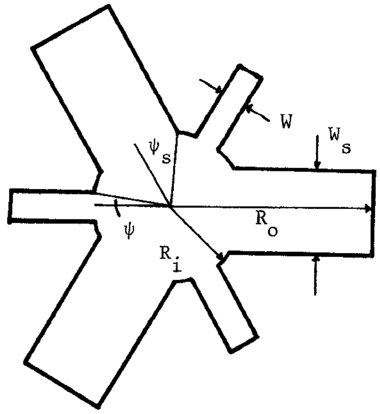


Fig. 1: Schematic diagram of a 3 port circulator consisting of 6 transmission lines alternately terminated in 50Ω lines and magnetic walls.

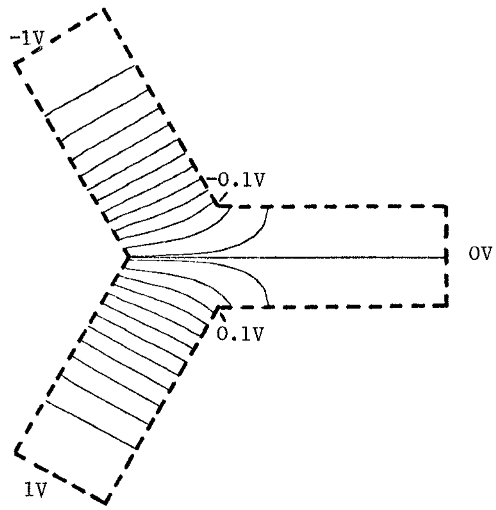


Fig. 2: Standing Wave Solution of WYE resonators.

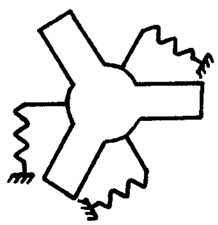


Fig. 3a
High Q connection

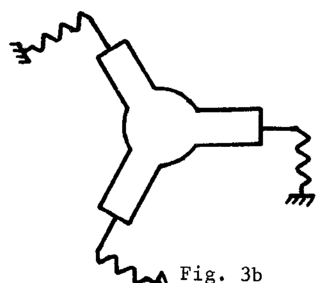
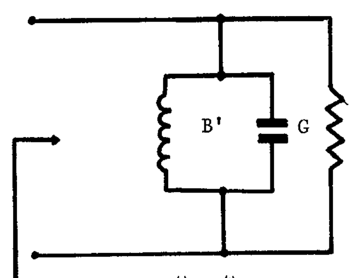


Fig. 3b
Low Q connection



$$G = \sqrt{3}\, B' \frac{\omega_{+1} - \omega_{-1}}{\omega_o}$$

Fig. 4: One port equivalent circuit of junction circulators.

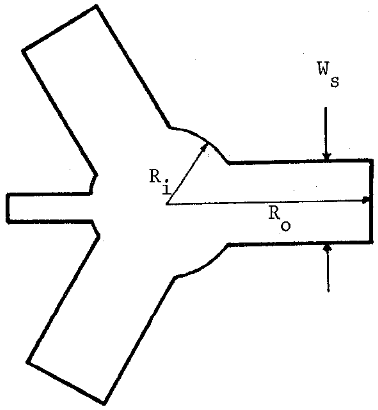


Fig. 5: Constituent WYE Resonator

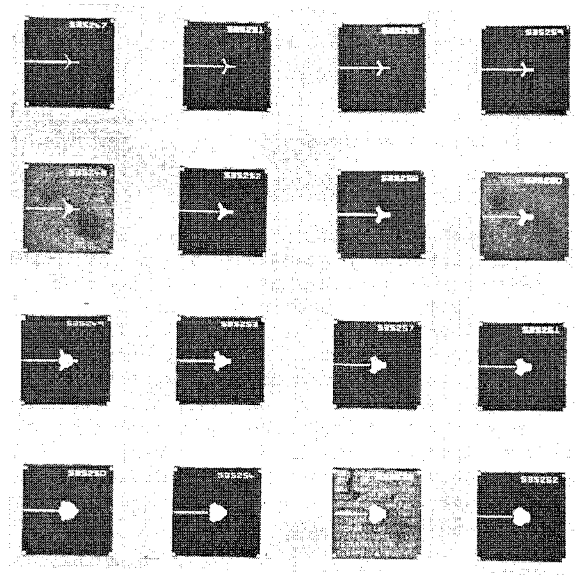


Fig. 7: Experimental Hardware

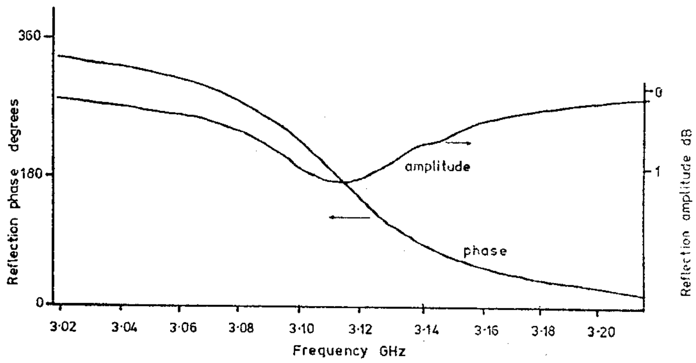


Fig. 6: Reflection coefficient of a constituent WYE Resonator

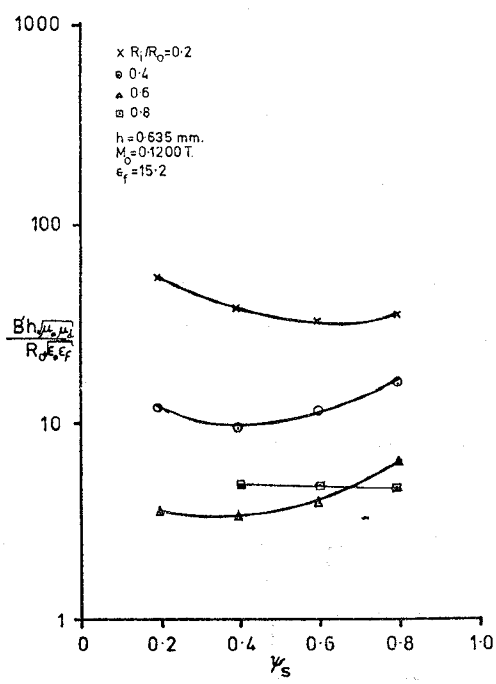


Fig. 8: Experimental Susceptance Slope Parameter

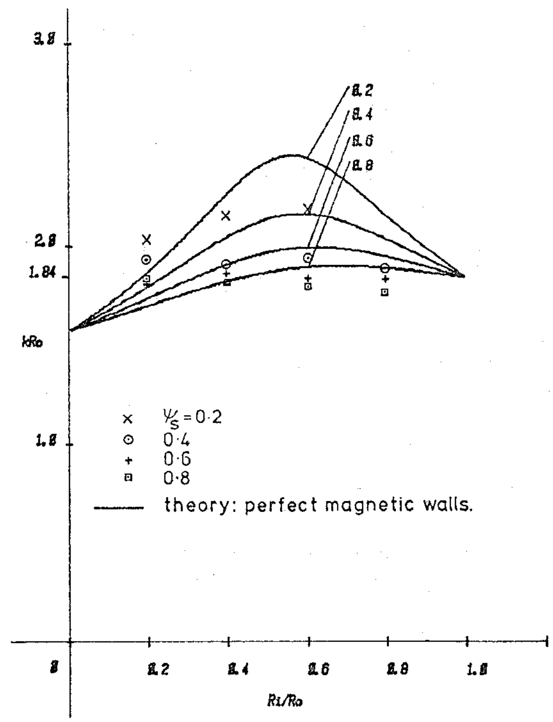
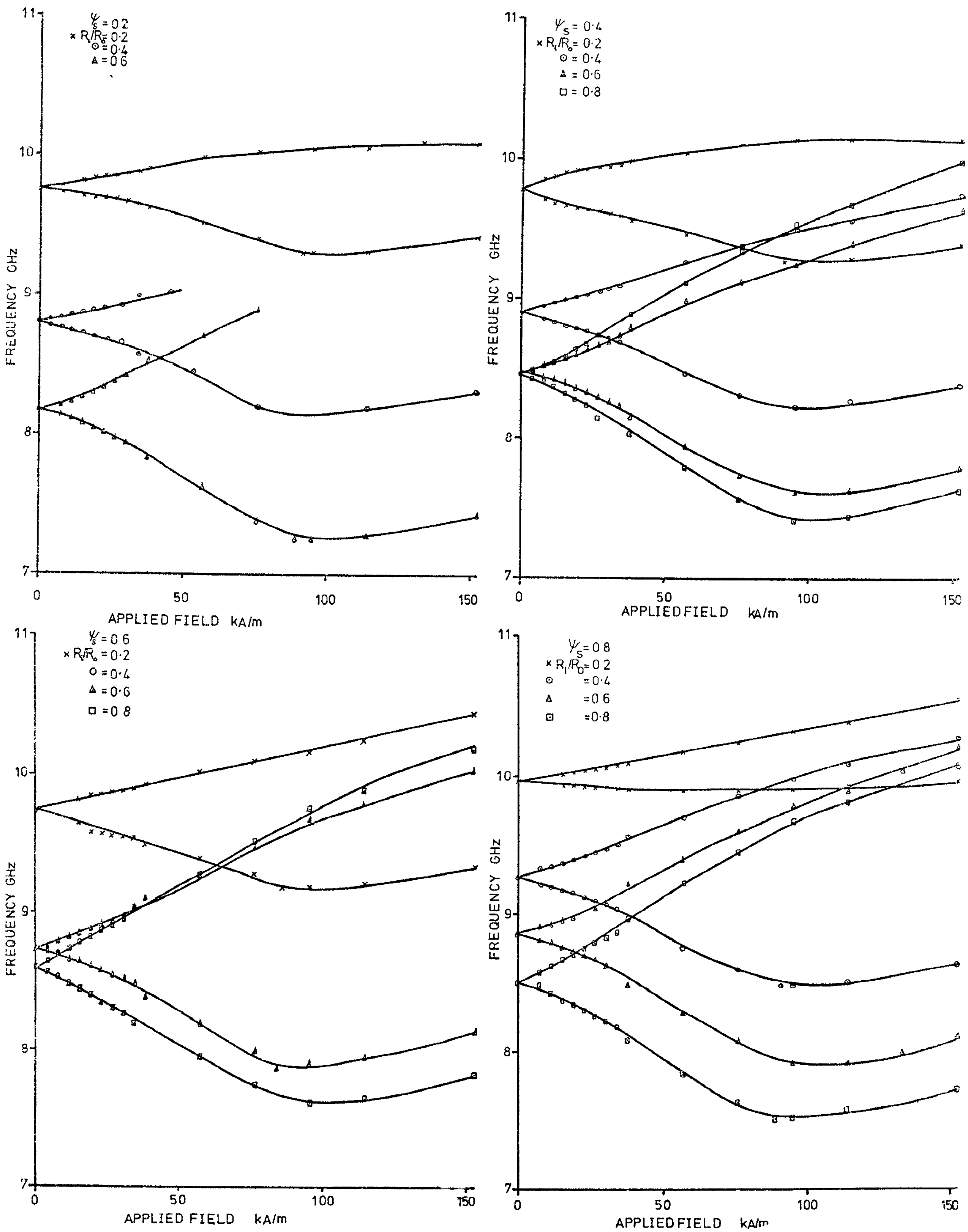


Fig. 9: Theoretical and experimental cut-off wave numbers



Figs. 10-13: Split frequencies of magnetised WYE Resonators